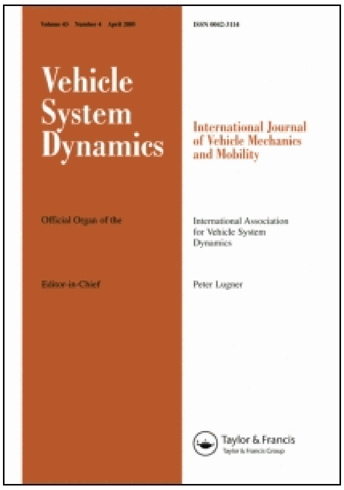

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Dynamic Coupling in Vehicles Under Automatic Control

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Dynamic Coupling in Vehicles Under Automatic Control

J. K. HEDRICK* and D. SWAROOP**

SUMMARY

This paper discussed the conditions under which longitudinal disturbances in strings of coupled vehicles can be made to attenuate as they travel upstream. The "slinky" effect is defined as the condition where these disturbances are amplified, a condition that can lead to collisions among upstream vehicles. Input/output linearization is applied to linearize the spacing error to throttle dynamics. Well known linear norm relationships are then applied to investigate the "slinky" effect. It is shown that broadcasting the velocity and acceleration of the lead vehicle to all vehicles in the string is sufficient to eliminate the slinky effect.

1. INTRODUCTION

Intelligent Vehicle/Highway Systems (IVHS) research is being vigorously pursued in many countries around the world. This research includes a wide variety of topics including traffic management systems, traveler information systems and advanced vehicle control systems. Current topics in advanced vehicle control systems include collision warning and avoidance systems, autonomous intelligent cruise control (AICC) [2] and fully automated platoons [4] or convoys. This paper concerns longitudinal spacing control laws that could be used in AICC systems where the driver is still in charge of steering or fully automated systems.

2. SPACING CONTROL

Figure 1 illustrates the definitions of inter-vehicle spacing, the distance between bumpers between the ith and i-1th vehicle is:

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$$\delta_i = x_{i-1} - x_i - l_i \tag{1}$$

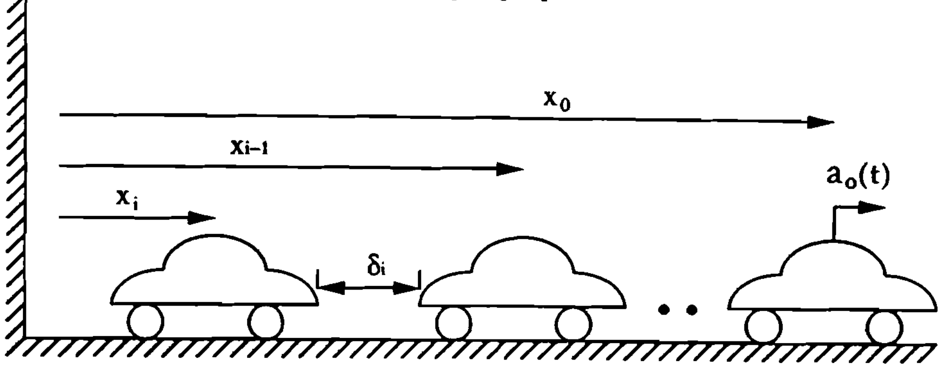

Figure 1. Distance Between Vehicles in String

We define the spacing error, ε_i, as:

$$\varepsilon_i = \delta_{i_d}(t) - \delta_i = \delta_{i_d}(t) - x_{i-1} + x_i + l_i \tag{2}$$

A simple vehicle dynamic model for a vehicle's longitudinal motion [4] is:

$$\dot{v}_i = \frac{1}{m_i}\left[r_1 T_{net}(\alpha_i, v_i) - r_2 T_L(v_i)\right] \tag{3}$$

The assumptions required to obtain equation (3) are:

- intake manifolds dynamics are very fast
- torque converter is locked
- pure rolling of the tires ($v_i = r_i^* \omega_e$)

The first assumption is that the mass air flow through the throttle equals the flow into the cylinders. The second assumes we are in a higher gear ratio where the torque converter is locked and the third ignores the slip at the tire/road interface. These assumptions imply that the vehicle's forward speed is directly related to the engine speed through the gear ratio, r_i^*. $T_{net}(\alpha_i, v_i)$ is the net combustion torque, α_i is the throttle angle, T_L is the load torque which includes rolling resistance, aerodynamics and gravity terms due to grade changes. The terms r_1 and r_2 are geometrical terms and gear ratios necessary to reflect all terms to the vehicle center of mass.

We can simplify (3) by defining a "synthetic" control, u_i, such that,

$$u_i \triangleq \frac{1}{m_i} [r_1 T_{net}(\alpha_i, v_i) - r_2 T_L(v_i)] \tag{4}$$

where we have assumed that the throttle can be varied such that,

$$T_{net}(\alpha_i, v_i) = \frac{1}{r_1} [m_i u_i + r_2 T_L(v_i)] \tag{5}$$

Equations (4) and (3) yield

$$\dot{v}_i = u_i \tag{6}$$

Returning to equation (2) we differentiate to get

$$\dot{\varepsilon}_i = v_i - v_{i-1} \tag{7}$$

$$\ddot{\varepsilon}_i = u_i - a_{i-1} \tag{8}$$

In equation (8) the desired spacing δ_{id} is assumed constant. At this point we have a linear relationship between the synthetic control u_i and the spacing error, ε_i, and many different control methods could be chosen to design, u_i (see for example [4]). Considering the fact that we have made many approximations and that there are a number of uncertainties in our model we will use the "sliding" control approach to design the throttle controller. First we define a "surface," s_i, that has two properties, the first being that $s_i \equiv 0$ implies $\varepsilon_i(t)$ will approach zero and second that $\dot{s}_i$ explicitly contains u_i. Such a surface is

$$s_1 \triangleq \dot{\varepsilon}_i + q_1 \varepsilon_i \tag{9}$$

Differentiating yields

$$\dot{s}_i = u_i - a_{i-1} + q_1\dot{\varepsilon}_i \tag{10}$$

A convenient choice of u_i that will make s_i go to zero exponentially is

$$u_i = a_{i-1} - q_1\dot{\varepsilon}_i - \lambda s_i \tag{11}$$

Substituting (11) and (9) into (8) yields

$$\ddot{\varepsilon}_i + (q_1+\lambda)\dot{\varepsilon}_i + q_1\lambda\varepsilon_i = 0 \tag{12}$$

Stability requires q_1 and $\lambda > 0$.

3. Slinky Effect

Equation (12) guarantees that $\varepsilon_i(t)$ will approach zero exponentially but says nothing about the "string" stability, i.e., the attenuation or amplification of disturbances upstream. References [5,6,7] address this problem for general linear control laws. In this paper we look at the use of sliding control and its modification to guarantee string stability.

The next section reviews some useful results from linear system theory [3]. If h(t) is an impulse response function of a causal, stable, linear time invariant system and $\hat{h}(s)$ is its Laplace transform and transfer function between the input, u(t), and the output, y(t), then

$$\| h(t) \|_1 \triangleq \int_0^\infty | h(t) | dt \tag{13}$$

$$\| y(t) \|_\infty \triangleq \sup_{t\geq 0} | y(t) | \tag{14}$$

Reference [3] shows that

$$\| y(t) \|_\infty \leq \| h(t) \|_1 \cdot \| u(t) \|_\infty \tag{15}$$

Now if we define $\varepsilon_i(t)$ as the output, y(t), and $\varepsilon_{i-1}(t)$ as the input, u(t), then the "slinky" effect or disturbance amplification upstream can occur if,

$$||h(t)||_1 > 1 \quad (16)$$

and cannct occur if

$$||h(t)||_1 \leq 1 \quad (17)$$

One can develop some useful relationships [5,6] for $||h(t)||_1$, e.g.,

$$||h(t)||_1 \geq |\hat{h}(0)| \quad (18)$$

In fact,

$$||h(t)||_1 \geq |\hat{h}(j\omega)| \ , \text{ for any } \omega \geq 0 \quad (19)$$

If h(t) does not change sign, then

$$||h(t)||_1 = |\hat{h}(0)| \quad (20)$$

We will develop the string transfer functions,

$$\frac{\hat{\varepsilon}_1(s)}{\hat{a}_o(s)} \triangleq \hat{g}(s) \quad (21)$$

and

$$\frac{\hat{\varepsilon}_i(s)}{\varepsilon_{i-1}(s)} \triangleq \hat{h}(s) \quad (22)$$

The first transfer function $\hat{g}(s)$ defines the disturbance of the lead vehicle's acceleration, $a_o(t)$, to the spacing error between the lead vehicle and the first trailing vehicle ($\varepsilon_1(t)$). As long as $\hat{g}(s)$ is not identically zero then some disturbance will be propagated upstream. $\hat{h}(s)$ will tell us whether the slinky effect can ($||h||_1 > 1$) or cannot ($||h||_1 \leq 1$) occur. It is straightforward to show from equations (8) and (11) that we have perfect decoupling and that

$$\hat{g}(s) \equiv 0 \ , \ \hat{h}(s) \equiv 1 \quad (23)$$

Thus the slinky effect will not occur.

Since we have made many approximations in our design we must investigate the robustness to model error, sensor noise, signal processing lags, etc. As a beginning we can approximate signal processing and computation delays as

$$u_{i_s} = a_{i-1} - q_1\dot{\varepsilon}_i - \lambda s_i \tag{24}$$

and

$$\tau\dot{u}_i + u_i = u_{i_s} \tag{25}$$

where τ represents the processing delay. Combining (20), (21), (8) and taking Laplace transforms yields

$$\frac{\hat{\varepsilon}_1(s)}{\hat{a}_o(s)} = \frac{-\tau s}{\tau s^3+s^2+(q_1+\lambda)s+q_1\lambda} \triangleq \hat{g}(s) \tag{26}$$

and

$$\frac{\hat{\varepsilon}_i(s)}{\hat{\varepsilon}_{i-1}(s)} = \frac{s^2+(q_1+\lambda)s+q_1\lambda}{\tau s^3+s^2+(q_1+\lambda)s+q_1\lambda} \triangleq \hat{h}(s) \tag{27}$$

Let $c_1 \triangleq q_1+\lambda$, $c_2 \triangleq q_1\lambda$. It is straightforward to show that

$$|\hat{h}(j\omega)|^2 = 1 + \frac{2\tau\omega^2(c_1-\tau\omega^2)+\tau^2\omega^4}{(c_1-\tau\omega^2)^2\omega^2+(c_2-\omega^2)^2} \tag{28}$$

For any ω such that $c_1-\tau\omega^2 > 0$ we have $|\hat{h}(j\omega)| > 1$. Thus by equation (23), $\|h(t)\|_1 \geq |\hat{h}(j\omega)| > 1$. Therefore there exists a perturbation, $a_o(t)$, which will produce the slinky effect.

In an attempt to eliminate the slinky effect, we redefine the sliding surface (equation (9)):

$$s_i = \dot{\varepsilon}_i + q_1\varepsilon_i + q_2(v_i-v_o(t)) \tag{29}$$

In order to make $\dot{s}_i = -\lambda s_i$ we define our synthetic control, u_i, to be

$$u_i = \frac{1}{1+q_2}\left[a_{i-1}+q_2a_o(t)-(\lambda+q_1)\dot{\varepsilon}_i-\lambda q_1\varepsilon_i-\lambda q_2(v_i-v_o(t))\right] \tag{30}$$

First it is necessary to show that the control law (equation (30)) results in $\lim_{t\to\infty} \varepsilon_i(t) = 0$. u_i was chosen such that $\dot{s}_i = -\lambda s_i$, thus we know that

$$s_i(t)=s_i(0)e^{-\lambda t} \tag{31}$$

For i = 1, we have

$$s_1 = \dot{\varepsilon}_1 + q_1\varepsilon_1 + q_2\dot{\varepsilon}_1 = (1+q_2)\dot{\varepsilon}_1 + q_1\varepsilon_1 = s_1(0)e^{-\lambda t}$$

or

$$\dot{\varepsilon}_1 + \frac{q_1}{1+q_2}\varepsilon_1 = s_1(0)e^{-\lambda t} \tag{32}$$

Clearly if q_1, q_2 and λ are positive, then both ε_1 and $\dot{\varepsilon}_1$ go to zero as $t \to \infty$. Equation (29) for i = 2 can be expressed as

$$(1+q_2)\dot{\varepsilon}_2 + q_1\varepsilon_2 = -q_2\dot{\varepsilon}_1 + s_2(0)e^{-\lambda t} \tag{33}$$

Since $\dot{\varepsilon}$ is a damped exponential it is clear that ε_2 and $\dot{\varepsilon}_2$ are also damped exponentials. By induction one can show that

$$\lim_{t\to\infty} \varepsilon_i(t) = 0 \quad , \quad i = 1,\ldots,n \tag{34}$$

Next we investigate the string stability (slinky effect) of the new control law (equation (30)). We will reconsider the effect of a "signal processing" delay (equation 25). u_{i_s} now becomes defined by equation (30). After some algebra we obtain

$$\frac{\hat{\varepsilon}_1(s)}{\hat{a}_o(s)} = \frac{-\tau s}{\tau s^3 + s^2 + \frac{(\lambda+q_1+\lambda q_2)s}{1+q_2} + \frac{\lambda q_1}{1+q_2}} = \hat{g}(s) \neq 0 \tag{35}$$

and

$$\frac{\hat{\varepsilon}_i(s)}{\varepsilon_{i-1}(s)} = \hat{h}(s) = \frac{\frac{s^2}{1+q_2} + \frac{(\lambda+q_1)s}{1+q_2} + \frac{\lambda q_1}{1+q_2}}{\tau s^3 + s^2 + \frac{(\lambda+q_1+\lambda q_2)s}{1+q_2} + \frac{\lambda q_1}{1+q_2}} \tag{36}$$

It is shown in reference [8] that for small τ that $h(t) = L^{-1}[\hat{h}(s)]$ does not change sign and thus equation (20) applies, i.e.,

$$||h(t)||_1 = |\hat{h}(0)|$$

and we see from equation (36) that $\hat{h}(0) = 1$, therefore $||h(t)||_1 = 1$ and the slinky effect **cannot** occur for any input, $a_o(t)$.

4. Simulation

In this section we simulate a five car platoon. The lead vehicle's velocity profile (v_o) is shown in figure 2. The trailing four vehicles are under automatic control using the throttle control law developed in section 2 (equation 5). The model used for the simulation is described in [4] and is not as restrictive as that used here. In particular the intake manifold dynamics, torque converter characteristics and tire slip were included.

Figure 3 shows the case where the sliding control law (equation (11)) is used without lead vehicle information and also there is no signal processing lag. Even though equations (23) tell us that no slinky effect should occur we can see from figure 3 that the maximum spacing error is **amplified** by the upstream vehicles. This is due to the different model used for the simulation. Figure 4 is the same case including a signal processing lag of τ = .05 seconds. Clearly the slinky effect is worse. The control sampling time is 50 ms.

Figure 5 shows the case of sliding control with lead vehicle information (equation (30)) but without a signal processing lag. The parameters q_1, q_2 and λ were chosen such that $q_1 = q_2 = \lambda = 1.0$. It is clear from figure 5 that the disturbance is **attenuated** by the upstream vehicles. Even the effect of the signal processing lag and model difference does not produce any amplification as shown in Figure 6.

5. Conclusion

It has been shown that individual vehicle stability and "string" stability are two different concepts. This fact has been know for some time. This paper showed how to use a robust nonlinear control method (sliding control) to guarantee both vehicle stability and string stability with and without the inclusion of lead vehicle information into the sliding surface definition. Numerical simulations substantiated the theory.

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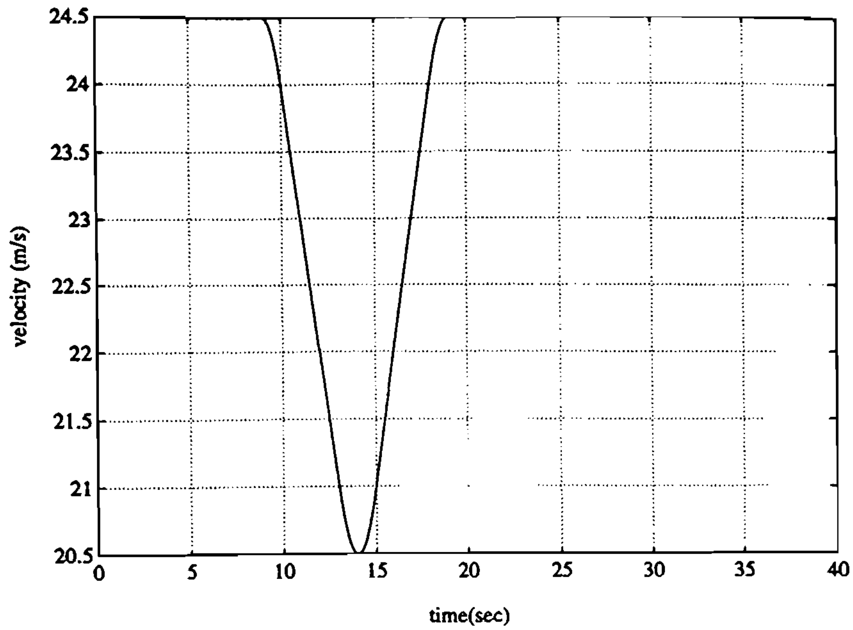

Fig. 2. Lead vehicle velocity profile.

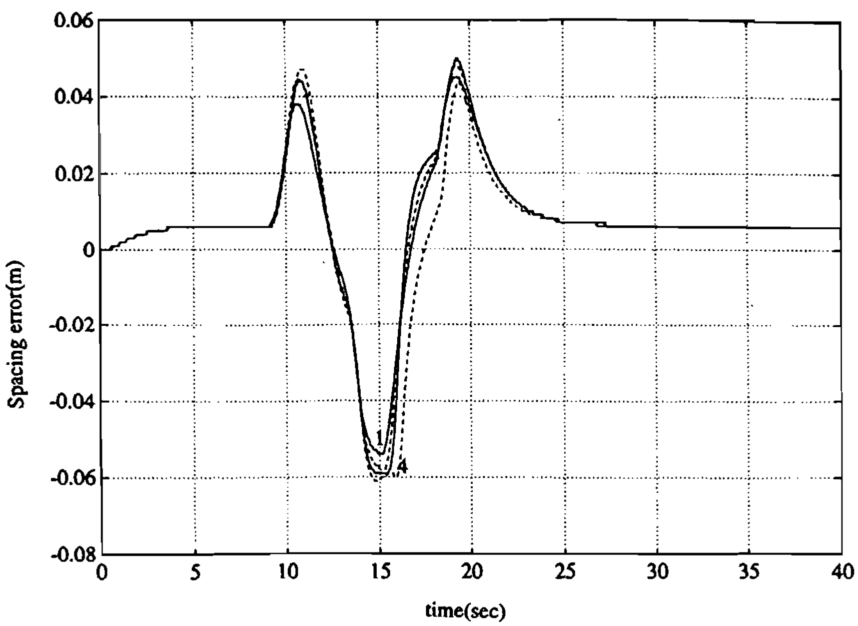

Fig. 3. Sliding control w/o lead vehicle info. and signal proc. lag.

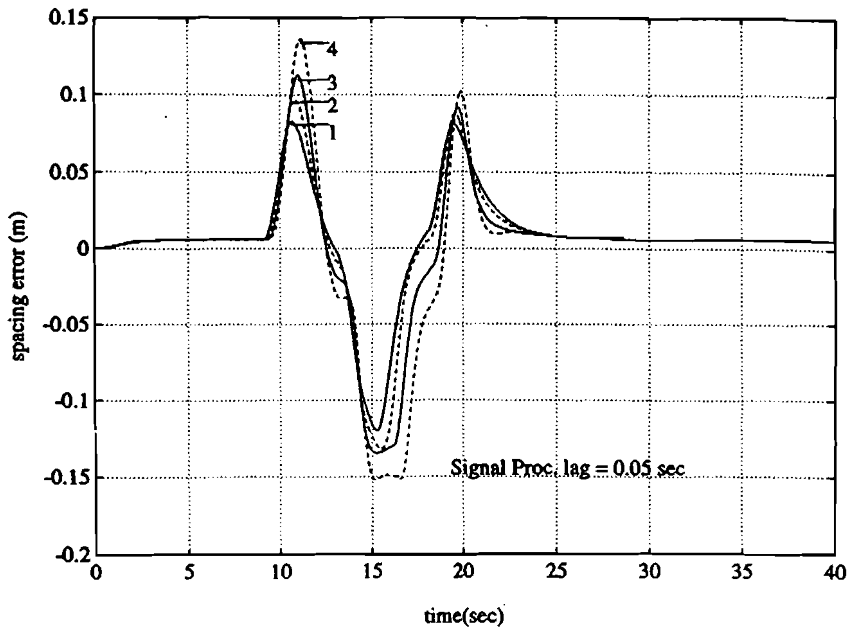

Fig. 4. Sliding control w/o lead vehicle info. and with signal proc. lag.

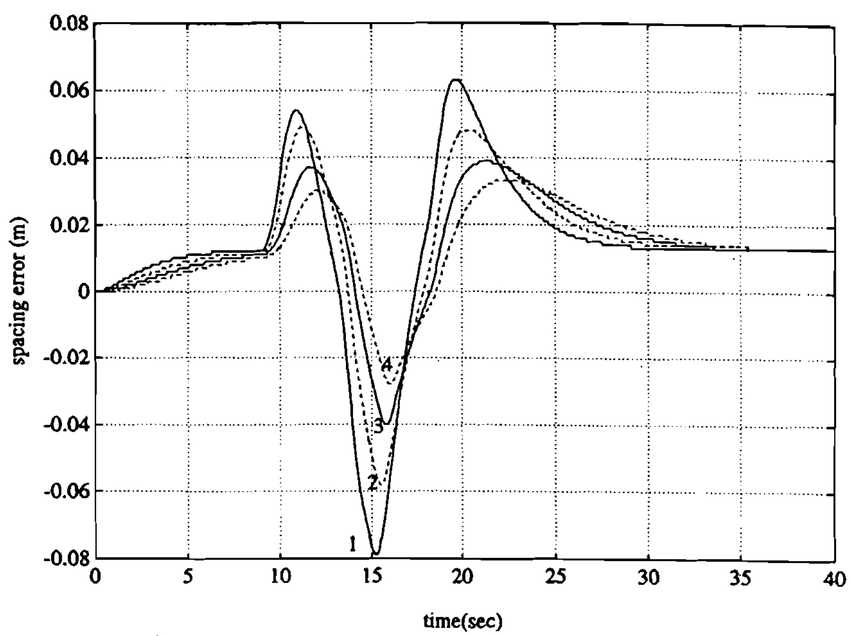

Fig. 5. Sliding control with lead vehicle info. and w/o signal proc. lag.

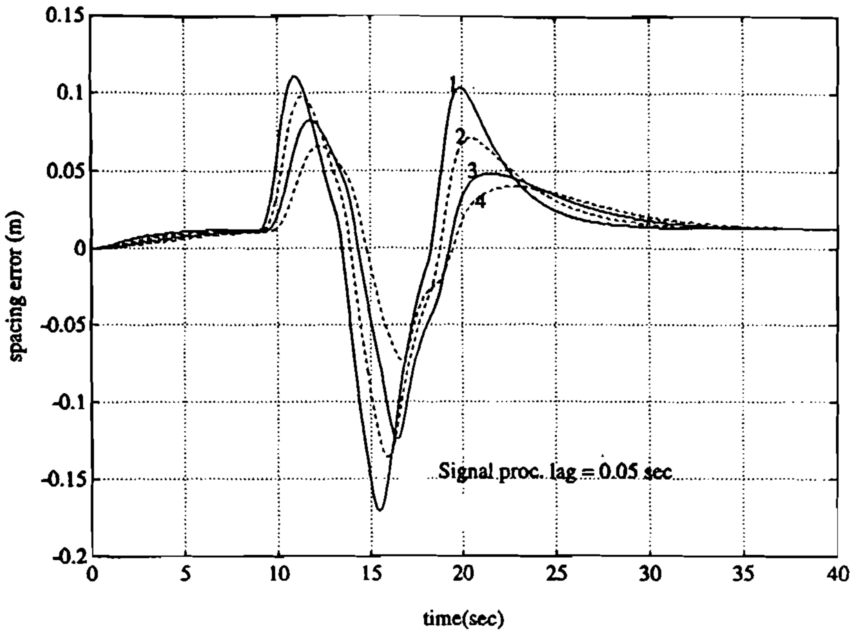

Fig. 6. Sliding control with lead vehicle info. and with signal proc. lag.